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A CHAOTIC AGRICULTURAL GROWTH MODEL

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Abstract

Chaos embodies three important principles: (i) extreme sensitivity to initial conditions; (ii) cause and effect are not proportional; and (iii) nonlinearity.

The basic aim of this paper is to provide a relatively simple agricultural growth model that is capable of generating stable equilibria, cycles, or chaos depending on parmeter values.

A key hypothesis of this work is based on the idea that the coefficient $\pi = \dfrac{\beta}{(\beta-\alpha)}$ plays a crucial role in explaining local agricultural stability, where α – coefficient of marginal labour productivity in agriculture; β – coefficient of average labour productivity in agriculture.

Key words: agricultural growth, stability, labour productivity, chaos

INTRODUCTION

Chaos theory is used to prove that erratic and chaotic fluctuations can indeed arise in completely deterministic models. Chaos theory reveals structure in aperiodic, dynamic systems. The number of nonlinear business cycle models use chaos theory to explain complex motion of the economy. Chaotic systems exhibit a sensitive dependence on initial conditions: seemingly insignificant changes in the initial conditions produce large differences in outcomes. This is very different from stable dynamic systems in which a small change in one variable produces a small and easily quantifiable systematic change.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983,1997,), Grandmont (1985), Goodwin (1990), Medio (1993,1996), Medio, A. and Lines, M (2004), Lorenz (1993), Shone, R.(1999) among many others.

The basic aim of this paper is to provide a relatively simple chaotic economic growth model · that is capable of generating stable equilibria, cycles, or chaos depending on parmeter values.

THE CHAOTIC GROWTH MODEL

Irregular time-path of the gross domestic product can be analyzed in the formal framework of the chaotic growth model :

$$\alpha = \frac{\Delta Y_d}{\Delta L} \tag{1}$$

$$\beta = \frac{Y_d}{L} \tag{2}$$

$$Y_d = (1-t)Y \tag{3}$$

$$Y_t = L_t^{1/2} \tag{4}$$

Where : Y – the real gross domestic product ; Y_d – disposable income ; L – labour force ; α – coefficient of marginal labour productivity ; β – coefficient of average labour productivity; t – tax rate. (1) determines the marginal productivity , α; (2) determines the average productivity, β ; equation (3) explains disposable income , Y_d ; equation (4) contains production function .

By substitution one derives:

$$Y_{t+1} = \frac{\beta}{(\beta-\alpha)} Y_t - \frac{\alpha\beta}{(1-t)(\beta-\alpha)} Y_t^2 \tag{5}$$

Further, it is assumed that the current value of the gross domestic product is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the economic growth rate depends on the current size of the gross domestic product, Y, relative to its maximal size in its time series Y^m. We introduce y as $y = Y/Y^m$. Thus s range between 0 and 1. Again we index y by t, i.e., write y_t to refer to the size at time steps t = 0,1,2,3,... Now growth rate of the gross domestic product is measured as

$$y_{t+1} = \frac{\beta}{(\beta-\alpha)} y_t - \frac{\alpha\beta}{(1-t)(\beta-\alpha)} y_t^2 \tag{6}$$

This model given by equation (6) is called the logistic model. For most choices of α , β, and t there is no explicit solution for (6). Namely, knowing α , β, and t and measuring y_0 would not suffice to predict y_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (6) can lead to very interesting dynamic behavior , such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of y_t. This difference equation (6) will posses a chaotic region. Two properties of the chaotic solution are important : firstly, given a starting point y_0 the solution is highly sensitive to variations of the parameters α , β, and t; secondly, given the parameters α , β, and t, the solution is highly sensitive to variations of the initial point y_0. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

LOGISTIC EQUATION

The logistic map is often cited as an example of how complex, chaotic behaviour can arise from very simple non-linear dynamical equations. The map was popularized in a seminal 1976 paper by the biologist Robert May. The logistic model was originally introduced as a demographic model by Pierre François Verhulst.

It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \pi z_t (1 - z_t), \quad \pi \in [0,4], \quad z_t \in [0,1] \tag{7}$$

is equivalent to the iteration of growth model (6) when we use the identification

$$z_t = \frac{\alpha}{(1-t)} y_t \quad \text{and} \quad \pi = \frac{\beta}{(\beta-\alpha)} \tag{8}$$

Using (8) and (6) we obtain

$$z_{t+1} = \frac{\alpha}{(1-t)} y_{t+1} = \frac{\alpha}{(1-t)} [\frac{\beta}{(\beta-\alpha)} y_t - \frac{\alpha\beta}{(1-t)(\beta-\alpha)} y_t^2] =$$

$$= \frac{\alpha \beta}{(1-t)(\beta-\alpha)} y_t - \frac{\alpha^2 \beta}{(1-t)^2(\beta-\alpha)} y_t^2$$

Using (7) and (8) we obtain

$$z_{t+1} = \pi z_t (1 - z_t) = \frac{\beta}{(\beta-\alpha)} \frac{\alpha}{(1-t)} y_t \left(1 - \frac{\alpha}{(1-t)} y_t\right)$$

$$= \frac{\alpha \beta}{(1-t)(\beta-\alpha)} y_t - \frac{\alpha^2 \beta}{(1-t)^2(\beta-\alpha)} y_t^2$$

Thus we have that iterating $y_{t+1} = \frac{\beta}{(\beta-\alpha)} y_t - \frac{\alpha \beta}{(1-t)(\beta-\alpha)} y_t^2$ is really the same as iterating $z_{t+1} = \pi z_t (1 - z_t)$ using $z_t = \frac{\alpha}{(1-t)} y_t$ and $\pi = \frac{\beta}{(\beta-\alpha)}$. It is important because the dynamic properties of the logistic equation (7) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that :

- (i) For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;
- (ii) For $1 < \pi < 3,57$ there exist fixed points the number of which depends on π;
- (iii) For $1 < \pi < 2$ all solutions monotnically increase to $z = (\pi - 1) / \pi$;
- (iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;
- (v) For $3 < \pi < 4$ all solutions will continously fluctuate;
- (vi) For $3,57 < \pi < 4$ the solution become "chaotic" wihch means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

CONCLUSION

This paper suggests conclusion for the use of the chaotic economic growth model in predicting the fluctuations of the population. The model (6) has to rely on specified parameters α , β, and t and initial value of the gross domestic product, y_0. But even slight deviations from the values of parameters α , β, and t and initial value of the gross domestic product, y_0, show the difficulty of predicting a long-term economic growth.

A key hypothesis of this work is based on the idea that the coefficient A key hypothesis of this work is based on the idea that the coefficient $\pi = \frac{\beta}{(\beta - \alpha)}$ plays a crucial role in explaining local economic stability , where α – coefficient of marginal labour productivity ; β – coefficient of average labour productivity.

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